ABLATION OF A METAL BODY NEAR THE STAGNATION POINT

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An approximate analytical solution is obtained for the ablation of a blunt metallic body near the stagnation point.

The steady ablation of a blunt body in the neighborhood of the stagnation point is considered, subject to the following assumptions. The influence of the molten film on heat transfer and friction in the boundary layer is neglected, since the flow velocity of the film is very much less than the velocity at the outer edge of the boundary layer. Thus, the heat flux and friction at the outer edge of the film can be taken as being the same as for a film at rest with interface temperature T_0. The molten film is assumed to be an incompressible liquid with constant thermophysical properties. The pressure distribution in the film is assumed to be Newtonian.

It follows from the results of [2] that the flow of the film in the vicinity of the stagnation point is absolutely steady. Under these assumptions, the system of equations describing conservation of fusion is analogous in form to that for a laminar boundary layer in an incompressible gas in the neighborhood of the stagnation point [3]. In the system of coordinates coupled with the gas-liquid interface and moving within the body at the breakdown rate w, the equations are:

$$(ux)_x + (vx)_y = 0,$$
$$uu_x + vu_y = \beta_2^2 x + \nu_2 u_{yy}, \qquad (1)$$
$$uT_x + vT_y = \varkappa_2 T_{yy}.$$

The symbols for velocity components and thermophysical parameters are the conventional ones. $\beta_2^2 = -(\rho_2 x)^{-1} P_x$ is the velocity gradient in the film.

The boundary conditions are as follows:

$$y = 0,\ v = 0,\ \mu_2 u_y = \tau_{10} = A_\tau x,\ \lambda_2 T_y = q_{10} = \alpha (T_l - T_0),\ y = -\delta,$$
$$u = 0,\ T = T_f. \qquad (2)$$

The values of the proportionality factor A_τ, in the expression for friction in the boundary layer, and the heat transfer coefficient α are known:

$$A_\tau = 0.763\,\mathrm{Pr}_1^{2/3} \sqrt{(\rho_1 \mu_1 \beta_1^3)_l},\ \beta_1 \equiv \frac{du_l}{dx} [-(\rho_l x)^{-1} P_x]^{1/2}, \qquad (3)$$

$$\alpha = 0.763\,\mathrm{Pr}_1^{-2/3} \bar{C}_p \left(\frac{\rho_0 \mu_0}{\rho_l \mu_l}\right)^{0,1} \sqrt{(\rho_1 \mu_1 \beta_1)_l}. \qquad (4)$$

The temperature distribution in the body for a given heat flux, and the temperature at the uniformly moving boundary were examined in [4], where the relationship between breakdown rate and heat flux q supplied to the body was obtained. For constant thermophysical parameters this takes the form:

$$q = \lambda_2 T_y(-\delta) = \rho_3 w Q_0, \qquad (5)$$

where $Q_0 = Q_f + C_3(T_f - T_{-\infty})$ is the heat required per unit mass to heat to the fusion temperature and liquefy. To the above boundary condition is added the law of conservation of mass on passing through the fusion front.

$$\rho_2 v(-\delta) = \rho_3 w. \qquad (6)$$

Thus, the problem reduces merely to that of solving a system of equations for fusion.

Reduction of the initial system of equations to ordinary differential equations:

We seek a solution in the form

$$u = \beta_2 x f'(z),\ v = -2\sqrt{\beta_2 \nu_2} f(z),\ T = T_f\, \theta(z),\ z = \sqrt{\frac{\beta_2}{\nu_2}}\, y. \qquad (7)$$

Substituting (7) in (1) and boundary conditions (2), (5), (6), we arrive at the following boundary value problem:

$$f'^2 + 2ff'' = f''' + 1. \tag{8}$$

$$\theta'' + 2\mathrm{Pr}_2 f\theta' = 0. \tag{9}$$

$$z = 0,\ f = 0,\ f'' = b,\ \theta_0' = \frac{\alpha}{\lambda_2}\left(\frac{\nu_2}{\beta_2}\right)^{1/2}(\theta_l - \theta_0), \tag{10}$$

$$z = -\left(\frac{\beta_2}{\nu_2}\right)^{1/2}\delta \equiv a,\ f' = 0,\ \theta = 1,\ \theta_a' = -2\mu_2\lambda_2^{-1} T_f^{-1} Q_0 f(a), \tag{11}$$

where

$$\theta_l = T_l/T_f,\ \theta_0 = T(0)/T_f,\ b = A_\tau \nu_2^{1/2}/\mu_2\beta_2^{3/2}. \tag{12}$$

The nonlinear boundary problem (8)-(11) contains two unknowns θ_0 and a. Eliminating θ_0 from the boundary conditions, from the last relation in (11) we obtain the relationship needed to determine the unknown film thickness, which completes the problem. Integrating the energy equation, we have

$$\theta' = \theta_0' \exp\left(-2\mathrm{Pr}_2 \int_0^z f dt\right) \tag{13}$$

and

$$\theta = 1 + \theta_0' [J(z) - J(a)], \tag{14}$$

where

$$J(z) = \int_0^z \exp\left(-2\mathrm{Pr}_2 \int_0^t f d\lambda\right) dt. \tag{15}$$

The film surface temperature and the heat flux are determined from the formulas

$$\theta_0 = 1 - \theta_0' J(a), \tag{16}$$

$$\theta_0' = \alpha\left(\frac{\nu_2}{\beta_2}\right)^{1/2}(\theta_l - 1)\left[\lambda_2 - \alpha\left(\frac{\nu_2}{\beta_2}\right)^{1/2} J(a)\right]^{-1}. \tag{17}$$

Using (13), and taking into account (17) from boundary condition (11) for θ_a', we find the condition which closes the problem

$$-2(\beta_2\nu_2)^{1/2}\rho_2 Q_0 f(a) = [\alpha T_f(\theta_l - 1)]\lambda_2[\lambda_2 - \alpha(\nu_2/\beta_2)^{1/2} \times \times J(a)]^{-1} \exp\left(-2\mathrm{Pr}_2\int_0^a f dz\right). \tag{18}$$

The first term on the right in (18) is the thermal flux to the surface at the fusion temperature, while the second and third terms reflect the reduction of heat flux due to superheating of the film and to heat being carried away by the film. Thus, the problem reduces to integration of (8) with three boundary conditions and the supplementary condition (18) to determine the unknown parameter a.

Calculation of ablation parameters:

The boundary problem (8), (10), (11), (18) has an approximate analytical solution in the form of the principal part of a Taylor series for function f in the region z = 0. This follows from the smallness of parameter a (a ~ 1) corresponding to the dimensionless thickness of the film.

To find the expansion of function f in series, a knowledge of f'(0) = k(a) is necessary. This may be determined by writing the expansion of f'(z) in the neighborhood of zero, and expressing the higher derivatives in terms of k(a) from (8). Then, satisfying the boundary condition for z = a, we arrive at the equation

$$k + ba + \frac{k^2 - 1}{2}a^2 - \frac{k(k^2 - 1)}{12}a^4 + \ldots = 0. \tag{19}$$

As the roots of (19) are still not convenient for investigation, let us assume that the roots of the equation of order n+1 differ only slightly from the roots of the equation of order n, in view of the rapidly diminishing terms of the expansion of f'(z). The positive root must be chosen, since $f'(z) \sim \partial u/\partial y > 0$. The expansion for f takes the form

$$f = kz + \frac{b}{2}z^2 + \frac{k^2 - 1}{6}z^3 - \frac{k(k^2 - 1)}{60}z^5 + \ldots \tag{20}$$

Solving the simultaneous system of algebraic equations (19) and (20), and using (18), a, k(a) and f(a) ~ w may be determined by successive approximations. The temperature profile is found from (16) and (17).

It should be noted that the value of f(a) obtained from (20) differs from the results of numerical integration of the boundary problem (8), (10), (11), and (18), when a = 2, by an amount less than 1% of f(a). This again confirms that the solution in the form of (20) gives a good approximation to the exact solution of the problem. For small Pr_2 numbers (fused metals), determination of breakdown parameters is greatly simplified.

The rate of fusion w is found from (18) to an accuracy of 3-5% independent of the remaining breakdown parameters, since superheating of the fusion zone and the carrying away of heat by the film are negligible, because of the low viscosity and high thermal conductivity.

$$w = \alpha (T_l - T_f)/\rho_3 Q_0. \tag{21}$$

For small Pr_2, the function $J(z) \approx z$, and the temperature profile in the fusion zone is nearly linear

$$T = T_f + \alpha\lambda_2^{-1} (\nu_2/\beta_2)^{1/2} (T_l - T_f)(z - a). \tag{22}$$

The film thickness is easily determined from (19) and (20), since the value of f(a) is known with great accuracy.

The dependence of the ablation rate on the M number can be approximated with high accuracy by the formula

$$w = CM^n \sqrt{P}. \tag{23}$$

The constant C depends on the properties of the material, the flowing medium, and the body geometry. The exponent n depends only slightly on the nature of the gas. For air, n = 3.3.

Because of the above-mentioned fusion properties of liquid metals, the rate of ablation on the lateral surface of the body, where laminar flow in the boundary layer and stability of the film are maintained, is given by

$$w = w_{cr} q(x)/q_{cr}. \tag{24}$$

The stability parameter of the film is the quantity $Re_0 = \beta_2 K x^2/\nu_2$, which, according to the results of [2], must not exceed 150. An expression for the heat flux to the lateral surface of the body was obtained in [5].

Ablation of a body decelerating in flight at constant altitude:

It is known that if a body of high thermal conductivity is placed in a stream of hot gas, the surface temperature reaches the value T_f in a time of the order of 10^{-3} to 10^{-2} sec, and thus the rate of ablation becomes steady practically instantaneously. Hence it follows that a quasi-stationary study of the problem can be used to calculate ablation under changing external conditions.

Let us examine the following problem. A body undergoing ablation in flight due to resistance of the medium decelerates in a distance L from velocity v_0 to velocity v_L. It is required to determine the ablation Δ. Assuming quasi-stationary ablation, we have

$$\Delta = \int_{t_0}^{t_L} w(t)\, dt = \int_{M_0}^{M_L} w(M) \frac{dt}{dM}\, dM. \tag{25}$$

The derivative dt/dM may be found from the equation of motion of the body, which, for a quadratic drag law, takes the form

$$\frac{dv}{dt} = -Bv^2, \quad B = \frac{C_x F \rho_1}{2m} \tilde{p}. \tag{26}$$

By virtue of the assumption of quasi-stationary ablation, we can use (23) for w(M). Substituting (23) in (25) and using (26), we have

$$\Delta = \frac{CM_0^{n-1} \sqrt{\tilde{p}}}{Ba^* (n-1)} \left[1 - \left(\frac{M_L}{M_0}\right)^{n-1}\right], \quad a^* = \frac{v}{M}. \tag{27}$$

Formula (27) was obtained on the assumption C_x = const, but the decrease in mass is small compared to the initial mass. The value of M_L is found from (26)

$$M_L = M_0 \exp(-BL). \tag{28}$$

It is interesting to investigate how the ablation depends on the pressure for given body mass and shape, flight distance L, and properties of the medium. It is evident that the dependence is not monotonic, since increase of pressure leads, on the one hand, to an increase in heat flux, and, on the other, to an increase in drag. The value of the pressure p_*, for which ablation of the body attains a maximum value, may be necessary for the conduct of laboratory experiments.

Examining (27) for an extremum, we find, for the value p_*, the formula

$$\tilde{p}_* = 2.512m/C_x F \rho_1 (n-1) L, \quad (29)$$

from which it is easy to see the dependence of $\tilde{p}_*$ on the parameters of the problem. Formula (29) does not include the dependence on M_0, since it was assumed above that the exponent n in (23) and the drag coefficient did not depend on the M number.

If the body breaks down with vaporization and chemical reactions, the dependence of the breakdown rate on pressure will be more complex. However, the method described may be used for a rough estimate of p_*.

NOTATION

C_x – drag coefficient, slightly dependent on M for $M \geq 2$ [6]; F – frontal area of model = $2\pi R^2$; ρ_1 – density of medium at pressure of $9.8 \cdot 10^4$ newton/m^2; m – mass of model; $\tilde{p} = p/9.8 \cdot 10^4$ newton/m^2 – dimensionless pressure.

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